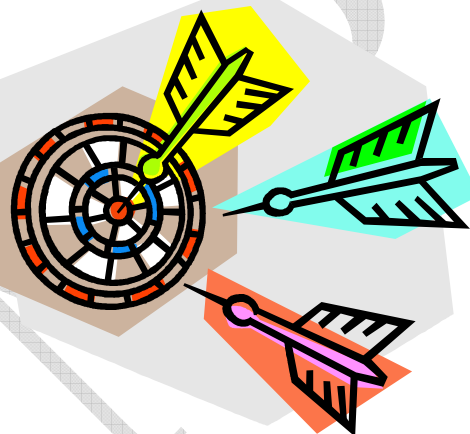




What's My Chance of Winning?



MATHEMATICS

GRADE 7

Item # 7.1.03

TEACHERS GUIDE

What's My Chance of Winning?

Mathematics

Grade: 7

OVERVIEW / CONTEXT

This task correlates a study of probability and ratio by having student's design, test, and evaluate their own game of chance. The teacher will lead the students in a discussion of probability and how it relates to games of chance that might be found at a school fair. The students will then randomly draw a game that they will design, explain, and test after approximately 1 week of in class instruction on probability. This game should emphasize one variable that may be manipulated to obtain a desired outcome. The students will design a table that will allow the students to test their game and modify it accordingly. The table will reflect: the variable being manipulated, the wins, losses, and closeness to the desired probability. The students will be individually assessed on a teacher-designed game. The student will be given: the cost of the prize, the materials, and the expected number of participants. The student will describe which variable to modify and show all mathematical computations to arrive at a desired income: cost ratio.

STANDARDS:

Probability

7.7.02 Formulate an argument on the likelihood of something happening or not happening in real life situations.

7.7.03 Describe, create, and solve real life and mathematical problems that involve probability.

Algebra

9.7.04 Solve multi-step equations involving integers using inverse operations and check the solutions for accuracy.

9.7.13 Create and interpret expressions, equations or inequalities and apply these algebraic methods to solve real life mathematical problems.

ACTIVITIES: 3

RECOMMENDED USES:

Diagnostic

Formative/Learning

Summative

VOCABULARY: probability, theoretical, empirical, ratio, proportion, income, cost

MATERIALS: Students will be designing and building their own games, therefore a lot of materials will have to be provided by the individual teams. The teacher will need an area large enough to set up the games of chance and an adequate paper supply for the copying of instructions and tests.

APPROXIMATE TIME: 4 – 50 Minute Sessions

TEACHER GUIDE

TITLE: PROBABILITY OF A GAME OF CHANCE

ACTIVITY 1



The school board has stated that it will approve games of chance at the annual student fair/community festival. You can earn money for your school related sport or activity by designing and operating a game.

Develop a game that you will use in the fair. You must determine what you will charge an individual to play the game in order to ensure a reasonable profit allowing for the cost of the prizes. Based upon this calculation you can determine the required probability of winning. Assume that 500 people will play your game (or a number of players approved by the class).

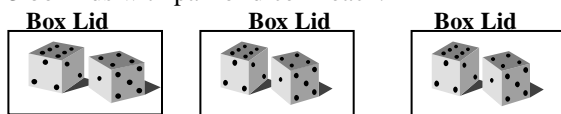
Submit the following in paragraph form to have your game approved:

- a) A detailed description of how to play the game.
- b) One aspect of the game that can be manipulated to affect the probability of winning with an explanation of how it will affect the probability.
- c) A diagram of the game.
- d) A list of required materials and costs even if the cost is zero.
- e) The prize for each winner and what it will cost.
- f) What each individual will be charged to play.
- g) Explain how you will meet the cost to income ratio of 1 to 3 (include calculation for: total cost, total income, and number of winners.
- h) The required probability of winning.

EXEMPLARY ANSWER: (Answers may vary depending on game selected)

“Roll of the Dice Game”

3 box lids with pair of dice in each.



People will have to roll 2 dice to try to get a certain number. We will manipulate the number that they have to get on the dice to effect the probability of winning because some combinations are more likely than others are.

Example: A result of “two” can be made only from $1 + 1$.

A result of “seven” can be made from $2 + 5$,
 $4 + 3$, $6 + 1$

Individuals will be charged 50 cents per roll of the dice. If they get the predetermined number they will win a \$5 gift certificate to the “time-out” arcade. We will have 3 trays of dice so those 3 people can play at a time.

PEOPLE EXPECTED TO PLAY: 500

STANDARDS / BENCHMARKS:

7.7.02

7.7.03

Reasoning - Embedded

TEACHER INSTRUCTIONS:

Students will be designing a game of chance that could be used as a fundraiser at a school fair. The students will need an understanding of what it means to find the probability of an event and the concept of setting up ratios and solving proportions.

This activity will require students to construct their own game of chance. Therefore, the students will supply the materials. This first activity will take one day.

The first activity will be introduced by a teacher led discussion regarding probability as ‘chance’, which will lead into a discussion of games of chance. The instructor will lead the class in a brainstorming activity that will generate a list of chance games that could be used as a fundraiser at a school fair. As a class, the issue of cost versus income will be addressed to decide an acceptable ratio of cost to income. The instructor may wish to use the ratio of **1:3** for cost to income if not, the mathematics can be adjusted to use any other ratio chosen. The teacher will then divide the class into teams of two or three to choose and design a game of chance.

The teacher may wish to write the names of the games of chance on a piece of paper, place them in a container and remove them and assign them to teams at random. This will eliminate duplication and further illustrate the idea of chance.

NOTE: Some games will not have a theoretical probability (ex. Toss games), these should not be discouraged. Please refer to the separate set of instructions for Activity #2. To guide students in the development of their descriptive paragraphs use a check list like the following:

| Description | Student | Teacher |
|---|---------|---------|
| A. Detailed description of how to play the game. | | |
| B. One aspect of the game that can be manipulated to affect the probability of winning with an explanation of how it will affect the probability. | | |
| C. Diagram of the game. | | |
| D. List of required materials and cost even if the cost is zero. | | |
| E. Prize for each winner and what it will cost. | | |
| F. What each individual will be charged to play. | | |
| G. Explain how you will meet the cost to income ratio of 1 to 3 (include calculation for: total cost, total income, and number of winners. | | |
| H. Required probability of winning. | | |

TEACHER GUIDE

TITLE: WHAT'S MY CHANCE OF WINNING?

ACTIVITY 2

Games with a theoretical probability:

Calculate the theoretical probability of winning your game. Show all work. Then, using a working proto-type of your game, construct a probability table for your game to investigate the actual probability of winning. This table will display the following:

- a) A title identifying the name of the game.
- b) A total number of attempts.
- c) The number of wins
- d) The number of losses.

The total number of attempts should be at least three times the theoretical probability to get accurate results.

Summarize the results of your table in a paragraph. Include a comparison of the theoretical to actual probability. If they do not match, what might account for this?

Reread the original description of your game. Then rewrite it to reflect the changes you made to get the required probability from Activity 1.

Then submit a typed copy for formal approval.

Games with no theoretical probability:

Use a working proto-type of your game to test the effect of modifying the aspect of the game that you identified in Activity 1. Construct a probability table to record the results of your test. This table will display:

- a) A title identifying the game.
- b) The aspect of the game that is being manipulated to affect the probability of winning, and its value for each test.
- c) The total number of attempts.
- d) The number of wins.
- e) The number of losses.

You must keep changing the aspect of the game until the required probability of winning that you determined in Activity 1 has been reached. For each change in the aspect of the game, you must continue testing the game until you have two wins and then calculate the probability of winning. If it doesn't match the value required from Activity 1, then change the value of the aspect that you are modifying and test again. Every time the aspect of the game is changed, the probability table must show the corresponding number of wins and losses.

Summarize the results of your table in a paragraph. Include a discussion of the effect of the aspect of the game that you manipulated. Did it affect the game the way you thought it would? Were you able to achieve the required probability from Activity 1? What other aspects of the game could be manipulated to effect the probability, and would any of these have been easier to control?

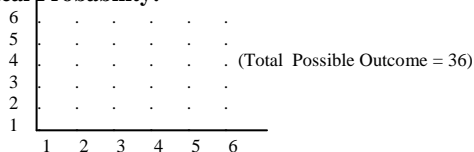
Reread the original description of your game. Then rewrite it to reflect the changes you made to get the required probability from Activity 1. Finally, submit a typed copy for formal approval.

Exemplary Response: (Paragraph)

The theoretical probability table showed that the ideal probability is $1/36$. The actual probability is determined by rolling the dice $4/108$ or $1/27$. Although the results may have been close, there is really nothing that can be said to influence the outcome. The number that was rolled for was 2. The revision of the game would include a new title: "Doing the Deuce" trying to roll the number 2.

Exemplary Response: (Answers may vary depending on game selected).

Theoretical Probability:



| # ON DICE | PROBABILITY | # ON DICE | PROBABILITY |
|-----------|--------------------|-----------|--------------------|
| 1 | Zero | 7 | 6 out of 36 = 1/6 |
| 2 | 1 out of 36 = 1/36 | 8 | 5 out of 36 = 5/36 |
| 3 | 2 out of 36 = 2/36 | 9 | 4 out of 36 = 1/9 |
| 4 | 3 out of 36 = 1/12 | 10 | 3 out of 36 = 1/12 |
| 5 | 4 out of 36 = 1/9 | 11 | 2 out of 36 = 1/18 |
| 6 | 5 out of 36 = 5/36 | 12 | 1 out of 36 = 1/36 |

STANDARDS/BENCHMARKS:

7.7.02

7.7.03

Reasoning - Embedded

TEACHER INSTRUCTIONS:

Students must be able to calculate the probability of an event, construct a probability table and determine actual probability that each group calculated and have them fix any errors. Students should collect materials and make their games at home. When they get to class they should be prepared to test their games. They will be working in the same teams of 2 or 3 students that they were in for Activity 1. This activity should take two days (or equivalent of 2 periods).

The types of games will fall into two categories:

- 1) Games that have a theoretical probability (no skill involved)
- 2) Games that do not have a theoretical probability (skill involved).

Any game that has some amount of skill involved will not have a theoretical probability, so the students will be determining only an empirical probability. Student activity sheets and rubric sheets have been developed for both cases:

If there is a theoretical probability, the students will calculate it and use it to finalize the aspects of the game so that they come as close as possible to attaining the required probability determined in Activity 1. For example, in a dice game they would have to choose what number to roll for to control the probability. Then they will do a test to determine actual probability.

If there is no theoretical probability (a game with skill involved) the students will modify a single aspect of the game through a series of tests until they attain the required probability that they calculated in Activity 1.

In both cases, the students will construct a probability table to record the results of their tests, summarize their findings and write a revised description of their game incorporating what they discovered in this activity.

We have chosen to have the people try to roll for double ones because our required probability was $1/30$ and the $1/36$ is as close as we could get. (We could have also chosen double 6.)

Actual Probability Test: Trying to Get Double Ones

| Attempt | Win | Loss |
|--------------|-----------------|------|
| 1 | | X |
| 2 | 1 st | |
| 3 | | X |
| 4 | | X |
| | | X |
| 105 | | X |
| 106 | 4 th | |
| 107 | | X |
| 108 (3 x 36) | | X |

TEACHER GUIDE

TITLE: PROBABILITY OF A GAME OF CHANCE

EVALUATIVE CRITERIA / RUBRIC – ACTIVITIES 1 & 2

| EVALUATIVE CRITERIA / RUBRIC – GAMES WITH A THEORETICAL PROBABILITY | |
|---|---|
| <ul style="list-style-type: none"> • Description and diagram are complete, clear and logically organized. Necessary changes have been made after testing the game. • Student identifies the variables affecting the problem, set up an equation and solved correctly, showed all steps. • Student identifies an aspect of the game that will affect the probability and correctly uses it to calculate theoretical probability and tests the required number of times to get actual probability. • Student includes title, labels columns correctly, table is neat, organized and uses the appropriate number of trials. Summary reflects the results of table and makes a valid comparison of aspect that was manipulated. | 3 |
| <ul style="list-style-type: none"> • Description and diagram are missing some attributes. Necessary changes have been made after testing the game. • Student identifies the variables affecting the problem, set up the equation, showed all steps but made computational errors. • Student identifies an aspect of the game that will affect the probability and correctly uses it to calculate theoretical probability but does not test the required number of times in determining actual probability. • Student includes title, labels columns correctly; table may have slight neatness and organization faults, uses appropriate number of trials. Summary reflects results of table and makes mostly valid comparison of aspect manipulation. | 2 |
| <ul style="list-style-type: none"> • Description and diagram are missing some attributes and necessary changes have not been made after testing. • Student identifies some variables, set up and solved part of the equation; work was incomplete and computed incorrectly. • Student identifies an aspect of the game that will affect the probability but makes computational errors in theoretical probability. • Student includes title, labels columns correctly; table is slightly less neat. Not enough trials are included. Summary reflects results of table, but the summary and discussion are incomplete. | 1 |
| <ul style="list-style-type: none"> • Description and diagram are incomplete so that game can not be tested. • Student does not identify and include variables affecting the problem. Equation not solved or major computational errors. • Student identifies an aspect of the game that does not affect the probability. • Student does not include the necessary parts of the table. Organization and neatness bad. Summary is incomplete and no comparison is made. | 0 |

| EVALUATIVE CRITERIA / RUBRIC – GAMES WITH NO THEORETICAL PROBABILITY | |
|---|---|
| <ul style="list-style-type: none"> • Description and diagram are complete, clear and logically organized. Necessary changes have been made after testing the game. • Student identifies the variables affecting the problem, set up an equation and solved correctly, showed all steps and had no computational errors. • Student manipulates an aspect of the game that does affect the probability and goes through testing until they win at least twice. Computation of probability is done correctly. • Student includes title, labels columns correctly, table is neat, organized and uses the appropriate number of trials. Summary reflects the results of table and makes a valid comparison of aspect that was manipulated. | 3 |
| <ul style="list-style-type: none"> • Description and diagram are missing some attributes. Necessary changes have been made after testing the game. • Student identifies the variables affecting the problem, set up the equation and solved, showed all steps but made computational errors. • Student manipulates an aspect of the game that does affect probability and goes through testing until they win at least twice. Computation of probability is done incorrectly. • Student includes title, labels columns correctly; table may have slight neatness and organization faults, uses appropriate number of trials. Summary reflects results of table and makes mostly valid comparison of aspect manipulated. | 2 |
| <ul style="list-style-type: none"> • Description and diagram are missing some attributes and necessary changes have not been made after testing. • Student identifies some variables, set up and solved part of the equation; work was incomplete and computed incorrectly. • Student manipulates an aspect of the game that does not affect the probability and goes through testing twice or only once. Computation of probability is done incorrectly. • Student includes title, labels columns correctly; table is slightly less neat. Not enough trials are included. Summary reflects results of table, but the summary and discussion are incomplete. | 1 |
| <ul style="list-style-type: none"> • Description and diagram are incomplete so that game can not be tested. • Student did not identify and include variables affecting the problem. Equation not solved or major computational errors. • An aspect of the game is manipulated that does not effect probability, goes through testing only once and computation of probability is done incorrectly. • Student does not include the necessary parts of the table. Organization and neatness bad. Summary is incomplete and no comparison is made. | 0 |

TEACHER GUIDE

TITLE: PROBABILITY OF A GAME OF CHANCE

ACTIVITY 3

You will be designing and operating a spinner game at the school fair for your class. A prize of a stuffed animal has already been chosen. Each stuffed animal cost \$3.25. You expect 500 people to play your game. The wood, paint and hardware for the spinner costs \$15.

Apply what you have learned in Activity #1, and Activity #2 to determine a combination of what you will charge an individual to play and probability of winning. Set up and solve a proportion to get a ratio cost to income of 1 to 3.

Show all work including: total cost, total income, number of winners, and probability of winning.

Summarize your results in a paragraph explaining how you will run the game. Include what you will charge each individual to play, the probability of winning that you calculated, and how you would design the spinner to attain the required probability of winning.

Exemplary Response:

A possible evaluation using a Spinner Game:

We will choose to charge 50 cents per spin.

Total cost = # of winners + prize cost + materials

Cost = $W(3.25) + \$15$.

Total Income = Number of players x cost to play

Income = $500 \times .50$

Ratio of Cost to Income:

$$(3.25w + 15.00)/(500 \times .50) = 1/3$$

$$(3.25w + 15.00)/250 = 1/3$$

Multiply both sides by the denominator or solve as a proportion:

$$3.15w + 15.00 = 83.3$$

$$3.25w = 68$$

$$w = 20.92 \approx 21$$

The number of players until someone wins must be 21 and the probability of winning is 1 out of 21.

To attain this probability of winning, I will divide the spinner circle into 21 spaces and mark one space "Winner".

Total cost = (# of winners) x (cost of 1 prize) + total materials
= $w(5) + \$1.50$

Total income = (# of players) x (cost to play)

$$= 500(0.50)$$

$$= 250$$

cost / income = $1/3 \quad ((w(5) + 1.50))/250 = 1/3$

$$250((w(5) + 1.50))/250 = 1/3(250)$$

$$w(5) + 1.50 = 83.3 - 1.50 = 81.83$$

$$((w(5))/5) = 81.93/5$$

$$w = 16.36$$

$$= 16 \text{ winners}$$

Probability of winning:

= number of winners/number of players

$$= (16/16) / (500/16) = 1/31.25 \approx 1/31$$

So one out of 31 people will win.

STANDARDS/BENCHMARKS:

7.7.02

7.7.03

9.7.04

9.7.13

Reasoning - Embedded

TEACHER DIRECTIONS:

This will be an individual evaluation of what the student has learned from both Activities #1 and #2.

This activity should be done on the last day of the probability unit, it will take one day.

Students will be given a situation where the game of chance, the cost of the prize, the cost of the materials and the expected number of people playing has already been determined. The student will have to determine a combination of the following:

1. Probability of winning.
2. What to change for an individual to play to maintain a pre-determined income: cost ratio.

The students will also have to describe how they will modify a variable in the game to obtain the desired probability of winning.

| EVALUATIVE CRITERIA / RUBRIC | |
|---|---|
| <ul style="list-style-type: none"> • Correctly identifies and manipulates an aspect to obtain the required probability, answer is correct. • Identifies and includes all variables affecting the problem, showed all steps and made no computational errors. | 3 |
| <ul style="list-style-type: none"> • Correctly identifies and manipulates an aspect to obtain the required probability, computational errors made • Identifies and includes all variables affecting the problem, showed all steps but made some computational errors. | 2 |
| <ul style="list-style-type: none"> • Incorrectly identifies aspect that was manipulated, computational errors made. • Identifies and includes most variables affecting the problem. Showed most steps and made some computing errors. | 1 |
| <ul style="list-style-type: none"> • No aspect identified or one that will have no affect at all. Mathematics incorrect or missing entirely. • Did not identify or include major variables affecting the problem, showed little work, computational errors. | 0 |